

Relationship between the Number of Tested Samples and the Estimation of SAG Milling Capacity Variability

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ABSTRACT

This study investigates the impact of sample size in the variability of estimating the capacity of a SAG mill. This research is based on an extensive database of over 800 samples gathered from a European mine and tested using the Geopyörä breakage device. The dataset includes crucial comminution parameters such as A_{xb} , DW_i , and BW_i .

This database was used to simulate hypothetical scenarios, with a different number of samples from the original database. Each scenario was submitted to the same procedure of estimating the mill's capacity. Ausenco's power-model AusGrind was used to calculate the specific energy (E_{cs}) from the circuit and mill. The results from the AusGrind were then used as inputs for a Monte Carlo simulation to perform the calculations of the SAG mill capacity.

The results of each scenario consisting of different sample sizes were then compared to analyse the impact of reduced number of samples on the variability of a SAG mill capacity.

INTRODUCTION

The determination of energy requirements and mill capacity is a crucial step in estimating the capital and operating costs of a comminution circuit, especially when working with autogenous (AG) and semi-autogenous (SAG) mills. Parameters obtained from ore characterization tests serve as a basis for mill sizing and are therefore essential for project success (Bailey et al, 2009). More specifically, AG/SAG mill sizing involves the use of mathematical models, the accuracy and reliability of which depend on the comprehensiveness and robustness of the ore characterization test campaign.

The objective of this study was to examine the impact of the number of samples on the variability of capacity and, consequently, on the performance of SAG mills installed in industrial circuits. To achieve this, the selected mathematical model for simulating an industrial SAG mill was initially calibrated and then using the Monte Carlo method, the mill capacity was then evaluated using different sample sizes from a large database obtained from a European mine.

METHODOLOGY

The database used comprised of the results of 821 tests conducted with the Geopyöra testing device (Bueno et al., 2021) on samples from the selected mine. Table 1 presents the three lithological groups found in the mine, whose names have been changed to generic terms.

Table 1 Distribution of the samples tested by the Geopyöra device

Ore Body	Number of samples	Measured parameters (average)				
		SG (t/m ³)	Drop Weight Index (kWh/m ³)	SD DWi	Bond Work Index (kWh/t)	SD BWi
Moderate	394	2.82	11.30	1.90	11.2	1.05
Hard	355	2.84	12.39	1.95	12.6	0.99
Very Hard	72	2.83	13.73	2.01	15.0	1.12
Overall	821	2.84	11.98	2.21	12.2	0.99

The parameters listed in Table 1 were used in a modified AusGrind power model (Lane, G. et al., 2013) developed for this work to calculate the specific energy of a SAG mill. The model was applied to ten different scenarios created for this study. The results from the power model were then used in a Monte Carlo simulation to analyze the variability on the SAG mill capacity.

Random Sample Selection Algorithm

A Python code was developed to simulate the specific energy required for the SAG mill based on the number of samples for each scenario. The purpose of the code was to randomly select a predefined number of samples from the database while maintaining the relationship presented in Table 2. This relationship was defined by the distribution of the groups presented in Table 1. The goal was to create similar conditions for each scenario, where the only difference between them would be the number of samples they comprise. The code was used to create ten scenarios, representing from 10 % to 100% of the total number of samples, while maintaining the relationship listed in Table 2.

Table 2 Distribution of ore types the original database

Ore type	Percent of samples
Moderate	48.0%
Hard	43.2%
Very Hard	8.9%

AusGrind model

The result of each sample within their respective individual scenario was simulated in the modified AusGrind model (Lane, G. et al., 2013), which utilizes different parameters and correction factors based on the type of circuit and equipment. However, since the focus of this study was the capacity of the SAG mill, only equation (1), described below, was employed.

$$SAG\ ecs \left(\frac{kWh}{t} \right) = (1.1 * DWi + 0.15) * 1.05 \quad (1)$$

Where SAG ecs represents the grinding specific energy required in kWh/t while the factor 1.05 is a correction factor related to the aspect ratio of the SAG mill used in this study.

Figure 1 depicts the graph illustrating the relationship between SAG ecs and Dwi (Lane, G. et al., 2013). As the original publication did not include the abscissa scale, this study adopted a maximum value of 14 kWh/t for SAG ecs, following Veillette and Parker, 2005. From this value, the corresponding DWi was determined and equation 1 was created using reverse engineering.

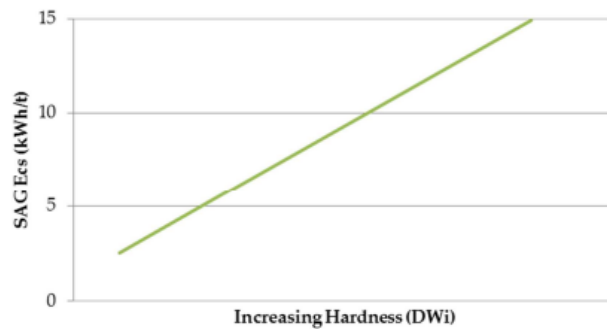


Figure 1 SAG mill base Ecs (Lane, G et al, 2013)

Table 3 displays the outcomes of the SAG ecs calculations for each instance and scenario, based on Equation 1. These outcomes represent an average derived from the calculated SAG ecs for each sample. Each case involves the replication of the same method, albeit with variations in random sample selection. As shown in Table 3, the effect of sample selection on the model is evident, as indicated by the variations across each case with an equal number of samples. Additionally, there is a convergence of the calculated mean towards a consistent value across all three cases as the sample size expands in each scenario.

Table 3 AusGrind SAG mill Ecs results

Scenario	Sample size	SAG mill Ecs (kWh/t)		
		Case 1	Case 2	Case 3
1	81	14.5	14.4	13.8
2	162	13.6	14.0	13.9
3	245	13.7	14.0	14.0
4	326	14.0	13.9	14.0
5	408	14.1	13.8	14.0
6	491	14.0	13.9	14.0
7	573	14.1	14.1	13.9
8	654	14.0	14.0	14.1
9	737	14.0	14.0	13.9
10	821	14.0	14.0	14.0

Monte Carlo Simulation

A Monte Carlo simulation method (Bueno et al, 2015) was conducted using a second Python code. The inputs for the code were the results of the SAG ECS calculation (Table 3), the standard deviation for each group, and the percentage of each ore type in the mill feed (Table 4). Those three parameters combined generated the SAG mill capacity for each scenario and case.

Table 4 Ore type distribution in the mill feed

Ore type distribution in the mill feed	
Moderate	31.0%
Hard	47.0%
Very Hard	22.0%

A normal distribution of the SAG mill ECS for each group was created and combined into a single distribution using a weighted average described by table 4, as resulted from the simulations. These results were used to calculate the mill throughput of solids in combination with the model developed by Morrell (1993) to determine the mill power draw. Using the model with the inputs from Table 5, the mill's operating power was determined to be 19.4 MW.

The results of the power consumption were used in equation (2) to obtain the mill throughput, where the denominator of the equation represents the average result for the ECS determined by the Monte Carlo simulation.

Table 5 SAG mill parameters

SAG mill design criteria	
Length (m)	6.7
Diameter (m)	11.6
Ball load (%)	15.0
Total load (%)	28.0
Speed (% critical)	78.0
Cone angle (°)	15.0
Concentration of solids of discharge slurry (%)	72.0

$$\text{throughput} \left(\frac{t}{h} \right) = \frac{\text{Mill power draw (kW)}}{\text{Mill specific energy} \left(\frac{kWh}{t} \right)} \quad (2)$$

The simulations and calculations were performed for each scenario of each case in this study.

RESULTS AND DISCUSSION

The simulation results were divided into two parts. The first was the analysis of the mill's capacity to compare the differences in capacity in each scenario and case, to assess the effect of reduced number of samples on mill capacity. The second part consisted in the analysis of the variability calculated from the results obtained by the Monte Carlo simulation, to assess the influence of a reduced number of samples in the variability of the mill capacity.

Throughput Analysis

The results from the Monte Carlo simulation were plotted on a graph using the x-axis to represent each scenario and the y-axis to represent SAG mill throughput calculations from Equation 2. The error bars on the graphs represent the 95% confidence interval calculated for each scenario with its respective number of samples. ¡Error! No se encuentra el origen de la referencia. illustrates the outcomes from the Monte Carlo simulation across all three cases for 10 scenarios, each one consisted in an increasing number of samples included in the simulations.

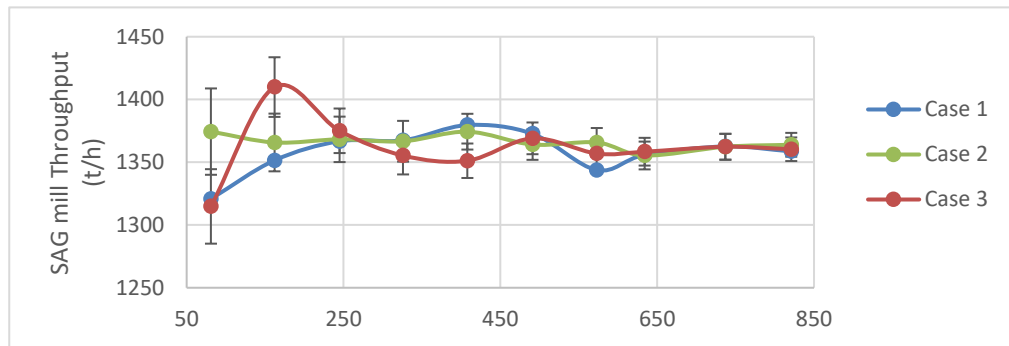


Figure 2 SAG mill throughput results

Cases 1 and 3 show similar results for the first scenario, the latter including the smaller number of samples, also case 3 has results from scenario two completely different from the ones obtained in scenario one, going from the lowest calculated throughput from scenario one to the highest one in scenario two. Case 2 showed the most uniform results when compared with the other two Cases, all

ten results of this case are within the 95 % interval of each other. The first two scenario results highlight the impact of a reduced number of samples on SAG mill throughput estimation, therefore indicating significant deviations from the base case, the latter consisting in the scenario that included the highest number of samples.

Scenario one of Case 1 and Case 3 do not encompass the values of scenario ten within their 95 % confidence interval because of the respective high differences. In a similar way the scenario two of Case 3 shows a significantly higher capacity than any other in this work, which is also higher than the 95% confidence interval of the base case as the number of samples increases with each scenario the results for all three cases progressively converge towards the expected value of scenario ten (higher number of samples). Furthermore, from scenario eight onwards, they exhibit the same results.

Variability Analysis of Mill Capacity

The second analysis involved studying the coefficient of variability of the results obtained through the Monte Carlo simulation. The main objective was to investigate how a reduced number of samples influences the variability of the mill capacity. **¡Error! No se encuentra el origen de la referencia.** presents the variability of the mill capacity results for each case. Here too, the 10 scenarios consisted in increasing number of samples included in the simulations.

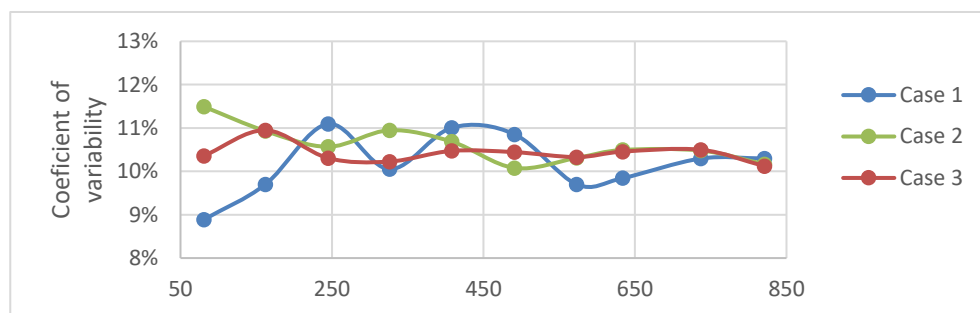


Figure 3 Variability results

The results of scenario one from case 1 showcases the lowest variability of this work due to the random selection algorithm picking samples with very similar characteristics, resulting in considerably smaller variation when compared to other results. However, the scenario three of case 1 is the second-highest calculated variability, as the algorithm selects samples with very diverse characteristics from one another, this showcases that a small sample size can drive the variability to both ends from the original result. On the other hand, cases two and three display consistent results for all scenarios, starting with a higher variability in the first scenario than the one displayed in scenario ten and as the sample size increases with each subsequent scenario, the variability gradually converges towards result of scenario ten. Analysing the curves for these cases suggests that

variability tends to be higher with smaller sample sizes, progressively approaching its actual value as the number of available samples increases.

CONCLUSIONS

The results obtained in the Monte Carlo simulations using the modified AusGrind method for the three studied cases showed that the capacity and variability of SAG mills are significantly influenced by the sample size and sample selection. In two out of the three cases, the estimated capacity using a small number of samples was considerably lower than that observed when using all available samples. It is thus concluded that a substantial number of samples is of paramount importance for assessing the variability in a SAG mill capacity.

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